Neutron Polarization at Small Scattering Angles Near an *s*-Wave Resonance

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It is shown that the neutron polarization by electromagnetic scattering at small scattering angles near an *s*-wave resonance may be observable at larger angles than considered previously. The angle increase possible is most pronounced at the interference minimum of the total-cross-section curve below the resonance energy. The nuclear spin-orbit polarization is also considered, and it is shown that for small-angle scattering and low energy and an assumed typical interaction strength this polarization is much less than the polarization arising from the electromagnetic spin-orbit interaction. This conclusion may be modified if the spin-orbit interaction strength is larger than that assumed because of a possible resonance behavior in the spin-flip scattering. The resonance behavior is indicated by a phase-shift analysis of the scattering, which also leads to the plane-wave Born-approximation result (nonresonant) under special conditions.

1. INTRODUCTION

IT has been shown by Schwinger[1] that fast neutrons scattered at small angles by nuclei can be polarized. The polarization arises from the interference between the nuclear scattering and the electromagnetic scattering. The angle of maximum polarization is nearly proportional to the nuclear charge but even for heavy nuclei the angle is of the order of a degree for 1 MeV. The maximum polarization approaches zero, however, as the neutron momentum approaches zero.

Margolis[2] pointed out that the polarization could still be large even for low neutron energy if the scattering occurred at the peak of a Breit-Wigner *s*-wave resonance. However, the effect was shown by Margolis to be a small angle effect as before. In fact in this case the angle of maximum polarization decreases with the neutron momentum as well as with nuclear charge.

The small-angle feature makes these polarization effects difficult to observe. The Schwinger-type effect was observed by Voss and Wilson[3] using 100-MeV neutrons scattered by uranium. The Margolis-type effect apparently has not been observed.

It is the main purpose of this paper to point out that large polarization effects may be observable at larger angles if the scattering occurs near an *s*-wave resonance. The angle of maximum polarization may be increased significantly if the scattering occurs at an energy where the nuclear scattering is suppressed, i.e., at energies where the cross section has a small value. This situation frequently occurs at energies just below the resonance energy where the nuclear potential and resonance scattering interfere destructively.

In addition to the electromagnetic spin-orbit interaction, neutrons experience a nuclear spin-orbit interaction which may change the features of the polarization in an experiment. Conditions where the nuclear and electromagnetic spin-orbit polarizations may be comparable are also considered here.

[1] J. Schwinger, Phys. Rev. **73**, 407 (1948).
[2] B. Margolis, Nucl. Phys. **22**, 498 (1961).
[3] R. G. P. Voss and R. Wilson, Phil. Mag. **1**, 175 (1956).

2. ELECTROMAGNETIC NEUTRON SCATTERING

A brief summary of the results obtained by Schwinger[1] and Margolis[2] will serve to introduce the essential features of the problem. Following Schwinger[1] the electromagnetic interaction H_e' between the incident neutron and the scattering nucleus is given by

$$H_e' = -\mu(e\hbar/2M^2c^2)\boldsymbol{\sigma}\cdot\mathbf{E}\times\mathbf{p}\,, \tag{1}$$

where $\mu=-1.91$ is the neutron magnetic dipole moment in nuclear magnetons; M, $\mathbf{p}$, and $\boldsymbol{\sigma}$ are the neutron mass, momentum, and Pauli spin, respectively; and $\mathbf{E}$ is the nuclear electric field. Schwinger[1] shows that the unscreened Coulomb field of a point-charge nucleus is a good approximation for scattering angles in the range

$$1/ka \ll 2\sin\tfrac{1}{2}\theta \ll 1/kR\,, \tag{2}$$

where θ is the scattering angle, a is the atomic screening radius, R is the nuclear radius, and $\mathbf{k}=\mathbf{p}/\hbar$ is the neutron wave number. Most cases of interest are for angles in this range.

For an incident plane wave ψ_{inc}

$$\psi_{\mathrm{inc}} = e^{i\mathbf{k}_0\cdot\mathbf{r}}\chi\,, \tag{3}$$

where $\hbar\mathbf{k}_0$ is the initial momentum and χ is a spin function, the asymptotic scattered wave ψ_{sc} in the direction of $\mathbf{k}$ is given by

$$\psi_{\mathrm{sc}} \sim (e^{ikr}/r) f(\theta)\chi\,, \tag{4}$$

where $f(\theta)$ is the scattering amplitude and $\mathbf{k}_0\cdot\mathbf{k}=k^2\cos\theta$. Using the plane-wave Born approximation for the electromagnetic scattering Schwinger[1] obtains

$$f(\theta) = f_0(\theta) - i\gamma\cot(\tfrac{1}{2}\theta)(\boldsymbol{\sigma}\cdot\mathbf{n})\,, \tag{5}$$

where $f_0(\theta)$ is the nuclear scattering amplitude,

$$\mathbf{k}_0\times\mathbf{k} = \mathbf{n}k^2\sin\theta\,, \tag{6}$$

$\mathbf{n}=-\mathbf{n}$ (Schwinger[1]) in accord with current convention and

$$\gamma = -\tfrac{1}{2}\mu(\hbar/Mc)(Ze^2/\hbar c)\,. \tag{7}$$

The polarization resulting from scattering an unpolarized incident wave is given by[1]

$$\mathbf{P} = -\mathbf{n}\frac{2\,\mathrm{Im}f_0(\theta)\gamma\cot\frac{1}{2}\theta}{|f_0(\theta)|^2+\gamma^2\cot^2\frac{1}{2}\theta} = P(\theta)\mathbf{n} \tag{8}$$

and the angle for maximum polarization θ_0 is given approximately by

$$\tan\tfrac{1}{2}\theta_0 = \gamma/|f_0(0)| \tag{9}$$

if $f_0(\theta)$ is well approximated by $f_0(0)$ for small angles. The polarization at the optimum angle θ_0 is then given by

$$P(\theta_0) = -\mathrm{Im}f_0(\theta_0)/|f_0(\theta_0)|\,. \tag{10}$$

Thus complete polarization occurs when $\mathrm{Re}f_0(\theta_0)=0$ but $\mathrm{Im}\,f_0(\theta_0)\neq 0$. Schwinger[1] gives the estimates

$$\begin{aligned}\tan\tfrac{1}{2}\theta_0 &= \gamma/R, \quad kR\ll 1,\\ &= \gamma/kR^2, \quad kR\gg 1,\end{aligned} \tag{11}$$

$$\begin{aligned}P(\theta_0) &= -kR, \quad kR\ll 1,\\ &= -1, \quad kR\gg 1.\end{aligned} \tag{12}$$

Since $\gamma/R = 1.1\times 10^{-3}ZA^{-1/3}$ it is clear that θ_0 is small. Even for uranium θ_0 is less than two degrees. However, the polarization drops off not too rapidly at angles larger than the optimum. The polarization is half the maximum polarization at an angle of about four times the optimum angle.

When the neutron scattering occurs near an s-wave resonance the s-wave nuclear scattering amplitude is given by[4]

$$f_0{}^0 = \frac{i}{2k}\left[1 - e^{-i2kR'}\left(1 - \frac{i\Gamma_n}{E-E_0+\frac{1}{2}i\Gamma}\right)\right], \tag{13}$$

where R' is the radius for potential scattering, Γ_n is the neutron resonance width, and Γ is the total resonance width.

Margolis[2] pointed out that for $kR\ll 1$ the scattering amplitude at the resonant energy is almost pure imaginary and gives complete polarization at the angle given by

$$\tan\tfrac{1}{2}\theta_0 = \gamma k\,. \tag{14}$$

Thus complete polarization can be obtained for $kR\ll 1$ but the optimum angle decreases below that indicated by Eq. (11).

Actually the condition $kR\ll 1$ is more restrictive than necessary. If $\mathrm{Re}f_0=0$ and $\mathrm{Im}f_0\neq 0$ then complete polarization occurs at the optimum angle given by Eq. (9). The real and imaginary parts of $f_0{}^0$ from Eq. (13) are given by

$$\begin{aligned}2k\,\mathrm{Re}f_0{}^0 = -[&(x^2-1+2\rho)\sin y\\ &+(1-\rho)2x\cos y]/(x^2+1)\,,\end{aligned} \tag{15}$$

$$\begin{aligned}2k\,\mathrm{Im}f_0{}^0 = 1+[&(-x^2+1-2\rho)\cos y\\ &+(1-\rho)2x\sin y]/(x^2+1)\,,\end{aligned} \tag{16}$$

with $x=2(E-E_0)/\Gamma$, $y=2kR'$, and $\rho=(\Gamma-\Gamma_n)/\Gamma$. If $\rho=0$ then one obtains that $\mathrm{Re}f_0{}^0=0$ when $\tan y=2x/(1-x^2)$ or $x=(-\cos y\pm 1)/\sin y$. In this case the optical theorem gives directly that $k^{-1}\,\mathrm{Im}f_0{}^0=(\mathrm{Im}f_0{}^0)^2+(\mathrm{Re}f_0{}^0)^2$ and if $\mathrm{Re}f_0{}^0=0$ then $\mathrm{Im}f_0{}^0=0$ or $\mathrm{Im}f_0{}^0=k^{-1}$. Thus, where constructive interference occurs (at resonance) one obtains

$$x_0{}^+=(1-\cos y)/\sin y\,,\quad \mathrm{Re}f_0{}^0=0\,,\quad \mathrm{Im}f_0{}^0=k^{-1}\,, \tag{17}$$

but where destructive interference occurs one obtains

$$x_0{}^-=-(1+\cos y)/\sin y\,,\quad \mathrm{Re}f_0{}^0=0\,,\quad \mathrm{Im}f_0{}^0=0\,, \tag{18}$$

where the $x_0{}^\pm$ are the zeros of $2k\,\mathrm{Re}f_0{}^0$ from Eq. (15). In this case the polarization by the optical theorem and Eq. (10) is given by $P=-(k\,\mathrm{Im}f_0{}^0)^{1/2}$. Hence at $x=x_0{}^+$ one obtains $P=-1$ while at $x=x_0{}^-$ one obtains $P=0$ when $f_0=f_0{}^0$.

On the other hand, if $\mathrm{Re}f_0=0$, $\mathrm{Im}f_0>0$ and $f_0(\theta)\approx f_0(0)$ then one obtains

$$\tan\tfrac{1}{2}\theta_0=\gamma/\mathrm{Im}f_0(0)\,, \tag{19}$$

$$P(\theta_0)=-1\,. \tag{20}$$

If the effects of $\rho>0$, nearby resonances, or higher angular momentum scattering contribute a small amount to f_0 then there will in general still be two values of x, x^+, and x^-, where $\mathrm{Re}f_0=0$ near the s-wave resonance energy E_0. But now, in general, $\mathrm{Im}f_0>0$ at both values. However, the value of $\mathrm{Im}f_0$ at x^- will usually be much smaller than its value at x^+ (near resonance). The energy region near x^- then is interesting because by Eq. (19) larger angles for maximum polarization are obtained there. If the angle θ_0 is small the following ratio follows from Eq. (19) and the optical theorem:

$$\theta_0{}^-/\theta_0{}^+\approx(\sigma^+/\sigma^-)(k^+/k^-)\,, \tag{21}$$

where σ is the total cross section and the $+(-)$ sign means evaluated at $x^+(x^-)$. Thus, if $\sigma^-\ll\sigma^+$, the optimum angle θ^- may be increased considerably beyond the value of θ^+ given by Eq. (14).

The use of the optical theorem in Eq. (19) in the form $\sigma=(4\pi/k)\,\mathrm{Im}f_0(0)$ to obtain Eq. (21) may be questioned since only the nuclear spin-independent amplitude is used. However it may be noted, first, that the total cross section contains only a very small contribution from the electromagnetic scattering from a screened Coulomb potential[1] and further the predominant part of this which occurs at small scattering angles is not included in the measurements of total cross section by the transmission method. On the other hand, the nuclear spin-orbit spin-flip contribution to the total cross section may or may not be small as discussed in the next section. Thus, Eq. (21) may be used only when this latter contribution to the total cross section is small. Of course when this is not the case, the whole preceding discussion must be modified to include the nuclear spin-orbit interaction as also is discussed in the next section.

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[4] H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. **96**, 448 (1954).

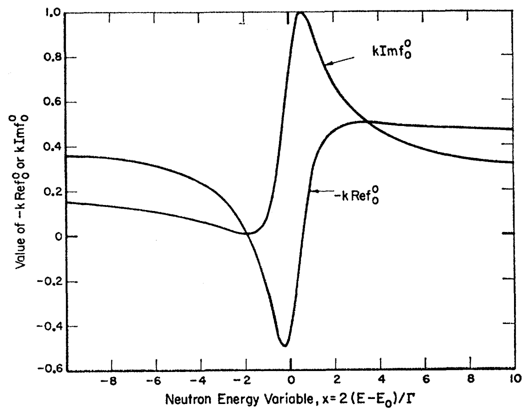

FIG. 1. The real and imaginary parts of $kf_0{}^0$ as a function of energy near a "pure" s-wave resonance ($\rho=0$) for $kR=0.5$.

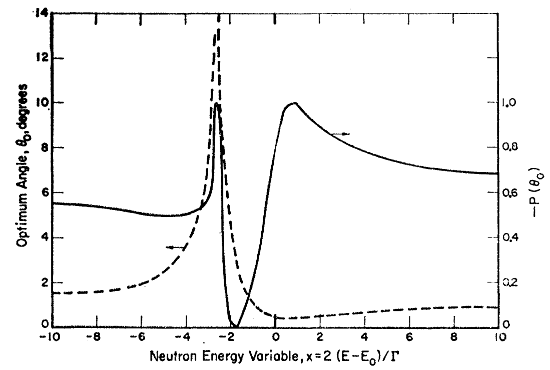

FIG. 3. The optimum angle for polarization θ_0 and the polarization $P(\theta_0)$ as a function of energy near an s-wave resonance ($\rho=0$) with 0.125 added to k Re$f_0{}^0$ for $kR=0.5$.

To illustrate these points the curves for $-k$ Re$f_0{}^0$ and k Im$f_0{}^0$ are shown in Fig. 1 for $y=1$ and $\rho=0$. The optimum angle for polarization θ_0 and its associated polarization $P(\theta_0)$ are shown in Fig. 2 as a function of x. If $0<\rho\ll 1$ the $x_0{}^{\pm}$ in Eqs. (17) and (18) may be replaced by $x^{\pm}/(1-\rho)$. However, the position where Im$f_0{}^0$ has a minimum is still at $x_0{}^{-}$ at which $2k$ Im$f_0{}^0$ $=\rho(1-\cos y)$ (this last result is valid for $0\leq\rho<1$).

The effects which may be observable when f_0 contains $f_0{}^0$ plus a small contribution from $l>0$ scattering can be seen in curves shown in Figs. 3 and 4 for $y=1.0$ and $\rho=0$. In Fig. 3 the effect of adding 0.125 to k Re$f_0{}^0$ is shown while in Fig. 4 the effect of subtracting 0.125 from k Re$f_0{}^0$ is shown. The latter case corresponds approximately to the p-wave contribution based upon the hard-sphere approximation for $kR=0.5$. Thus it is seen from Figs. 3 and 4 that the neutron energy range where large polarizations and large optimum angles occur together is rather sensitive to the contribution from $l>0$ scattering.

These considerations suggest that the enhanced optimum angle for polarization may be observable in a suitable experiment if one is careful and/or fortunate in the selection of the resonance. As discussed some further at the end of the next section the nuclei in the mass range $40<A<70$ appear to be interesting possibilities. A number of s-wave resonances are known[5] for some of these nuclei in the 1-keV–300-keV range which have significant interference minima in their total cross-section curve. Thus if as suggested by the previous discussion the real part of f_0 vanishes within the minima for some of these resonances, the enhancement of the optimum angle should be observable.

3. NUCLEAR SPIN-ORBIT INTERACTION

Besides the electromagnetic spin-orbit interaction neutrons also experience an effective nuclear spin-orbit interaction when scattered by nuclei. It is therefore of some interest to compare these two interactions because they both influence the neutron polarization upon scattering.

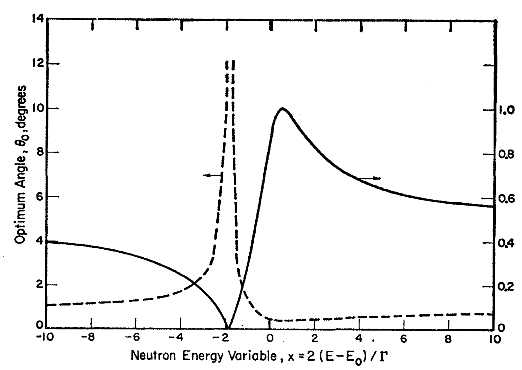

FIG. 2. The optimum angle for polarization θ_0 and the polarization $P(\theta_0)$ as a function of energy near a "pure" s-wave resonance ($\rho=0$) for $kR=0.5$.

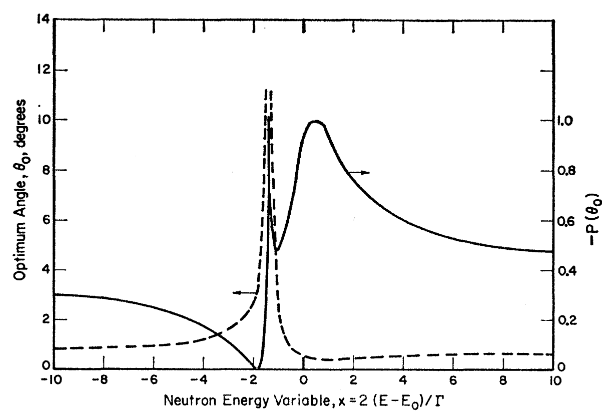

FIG. 4. The optimum angle for polarization θ_0 and the polarization $P(\theta_0)$ as a function of energy near an s-wave resonance ($\rho=0$) with 0.125 subtracted from k Re$f_0{}^0$ for $kR=0.5$.

[5] H. W. Newson, E. G. Bilpuch, F. P. Karriker, L. W. Weston, J. R. Patterson, and C. D. Bowman, Ann. Phys. (N. Y.) 14, 365 (1961).

The nuclear spin-orbit interaction H_n' for a spherical nucleus is usually given in the form

$$H_n' = V_{so}(R^2/r)(d/dr)q(r)\boldsymbol{\sigma}\cdot\mathbf{l}, \tag{22}$$

where the real part of the spin-independent nuclear potential $V(r)$ is given by[6]

$$\mathrm{Re}V(r) = -V_0 q(r) = -V_0\{1+\exp[(r-R)/a]\}^{-1},$$

with $\mathbf{l}$ the orbital angular momentum, "a" a diffuseness parameter, and V_{so} and V_0 are strength parameters. The V_{so} used here might typically have a value of about 0.3 MeV.

For a square-well ($a\to 0$) Eq. (22) simplifies to

$$H_n' = -V_{so}R\delta(r-R)\boldsymbol{\sigma}\cdot\mathbf{l}, \tag{23}$$

where $\delta(x)$ is the Dirac delta function. This form will be used in what follows to make estimates with $V(r)$ generally assumed to be a complex square-well potential.

Using Eq. (23) a plane-wave Born approximation for the effect of H_n' gives[7]

$$f(\theta) = iR\alpha(k/K)^2[(\sin KR/KR) - \cos KR]\sin\theta\boldsymbol{\sigma}\cdot\mathbf{n}, \tag{24}$$

where $K=2k\sin\frac{1}{2}\theta$ and $\alpha=2MV_{so}R^2/\hbar^2$. For $KR\ll 1$ Eq. (24) gives

$$f(\theta) = (iR/3)\alpha(kR)^2\sin\theta\boldsymbol{\sigma}\cdot\mathbf{n}. \tag{25}$$

It will be noted that the nuclear spin-orbit scattering amplitude as given by Eqs. (24) and (25) is imaginary like the electromagnetic amplitude but of opposite sign. Thus these effects interfere and using Eqs. (5) and (25) the two spin-orbit terms just cancel when

$$\sin\theta\tan\tfrac{1}{2}\theta = \frac{-3\mu}{4}\left(\frac{\hbar}{R_0Mc}\right)^3\left(\frac{Mc^2}{V_{so}}\right)\left(\frac{e^2}{\hbar c}\right)\left(\frac{Z}{A}\right)\left(\frac{1}{kR}\right)^2, \tag{26}$$

where $R=R_0A^{1/3}$. For $kR<1$ this critical angle is greater than about 20 deg. Viewed differently, at 5 deg the electromagnetic scattering amplitude is larger than the nuclear spin-orbit amplitude by a factor of about $16(kR)^{-2}$.

The use of the plane-wave Born approximation for the nuclear spin-orbit interaction may be questioned since α may be of the order of one. The Appendix gives a simple derivation of the exact phase shift expression for the nuclear spin-orbit interaction (delta function) in terms of the phase shifts without this interaction. The result obtained in the Appendix (keeping only p-wave terms and assuming $kR\ll 1$) for the spin-dependent amplitude is

$$kf(\theta) = \tfrac{1}{2}(e^{i2\delta_1^+} - e^{i2\delta_1^-})\sin\theta\boldsymbol{\sigma}\cdot\mathbf{n} \tag{27}$$

with

$$\tan\delta_1^{\pm} = \tan\delta_1^0[1-\alpha b_1^{\pm}(kR)^{-3}\tan\delta_1^0]^{-1}, \tag{28}$$

where the phase shift for $\alpha=0$ is given by $\delta_1^0(\eta=e^{i2\delta})$, and $b_1^+=1$, $b_1^-=-2$.

Equation (28) implies a resonance for $\tan\delta_1^{\pm}$ if the bracketed quantity is zero. If the phase shifts are small and $\tan\delta_1^0$ is approximated by the hard-sphere value, $-\frac{1}{3}(kR)^3$, for $kR\ll 1$, then Eqs. (27) and (28) lead to the plane-wave Born approximation result, Eq. (25), providing $\alpha\ll 1$. The real and imaginary parts of $f(\theta)$ may be written from Eq. (27) as

$$\mathrm{Re}kf(\theta) = \tfrac{1}{2}(\cos 2\delta_1^+ - \cos 2\delta_1^-)\sin\theta\boldsymbol{\sigma}\cdot\mathbf{n}, \tag{29}$$

$$\mathrm{Im}kf(\theta) = \tfrac{1}{2}(\sin 2\delta_1^+ - \sin 2\delta_1^-)\sin\theta\boldsymbol{\sigma}\cdot\mathbf{n}. \tag{30}$$

If α is not small compared to one but $\tan\delta_1^0 = -\frac{1}{3}(kR)^3$ and $kR\ll 1$ then Eq. (28) gives

$$\tan\delta_1^+ = -(kR)^3/(3+\alpha), \tag{31}$$

$$\tan\delta_1^- = -(kR)^3/(3-2\alpha). \tag{32}$$

Thus near $\alpha=\frac{3}{2}$, $\tan\delta_1^-$ is near resonance and may have a value in the range $-\infty<\tan\delta_1^-<\infty$. If $\tan\delta_1^-=\pm 1$ and $\tan\delta_1^+\approx 0$ then Eq. (30) gives $k\,\mathrm{Im}f(\theta)=\pm\frac{1}{2}\sin\theta\boldsymbol{\sigma}\cdot\mathbf{n}$ and now the magnitudes of the electromagnetic and the nuclear spin-flip scattering amplitudes are comparable at an angle given by

$$\sin\theta\tan\tfrac{1}{2}\theta = 2k\gamma \tag{33}$$

which for $\theta\ll 1$ gives

$$\theta(\mathrm{deg}) = 2.05[Z^2E(\mathrm{MeV})]^{1/4}. \tag{34}$$

Thus at lower energies if the nuclear spin-flip scattering has a p-wave resonance the polarization may be influenced by this resonance even at small angles. In addition to the resonance of the type just discussed for $\tan\delta_1^0=-\frac{1}{3}(kR)^3$ and $kR\ll 1$ there will also be a resonance in $\tan\delta_1^{\pm}$ near a resonance in $\tan\delta_1^0$ as can be seen from Eq. (28). The estimates given above serve to indicate the possible importance of the nuclear spin-orbit polarization for small angle scattering at low energy. Resonances for $l>1$ may also of course be important. Since

$$dY_{l,0}/d\cos\theta \underset{\theta\to 0}{\longrightarrow} [4\pi/(2l+1)]^{1/2}\tfrac{1}{2}l(l+1),$$

whereas

$$Y_{l,0} \underset{\theta\to 0}{\longrightarrow} [4\pi/(2l+1)]^{1/2},$$

then for small angles (nonzero) the relative importance of the larger angular momentum terms is greater for the spin-flip scattering than for the spin-constant scattering.

Equation (28) can also be considered in terms of the complex square-well expression for $\tan\delta_1^0$. In this case for $kR\ll 1$ one obtains[8]

$$\tan\delta_1^0 = -\tfrac{1}{3}(kR)^3(B-1)/(B+2) \tag{35}$$

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[6] P. A. Moldauer, Nucl. Phys. **47**, 65 (1963).

[7] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).

[8] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), 2nd ed., Chap. 5, p. 112.

with

$$B=-2+\beta j_0(\beta)/j_1(\beta), \quad (36)$$

where

$$\beta^2=2MV_0R^2/\hbar^2 \quad (37)$$

which is complex since V_0 is complex. In this case the resonances for $\tan\delta_1{}^0$ given by Eq. (35) are the well-known resonances in the p-wave strength function curve[6] while the resonances given by Eq. (28) give in principle the well-known splitting of the p-wave strength function resonances due to the nuclear spin-orbit interaction.

4. CONCLUSIONS

It has been shown that the small-angle electromagnetic scattering of neutrons near an s-wave resonance can lead to large polarizations at larger angles than those considered previously. This increase of the optimum angle for polarization arises because nuclear scattering is suppressed near the interference minimum of the resonance. The larger angles attainable are of course of some experimental interest.

The nuclear spin-orbit interaction has also been considered. It has been shown that for typical estimates of the interaction the nuclear spin-orbit polarization is small compared to the electromagnetic polarization at small scattering angles and low energy. However, because of a resonance behavior at somewhat larger values for the nuclear spin-orbit interaction the resulting polarization is rather sensitive to the interaction strength. The plane-wave Born approximation does not lead to this resonant behavior but a more detailed phase-shift analysis does and also leads to the plane-wave Born approximation under special conditions.

The nuclei in the mass range $40<A<70$ would appear to be good candidates for experiments to measure the polarization effects associated with Eq. (21). Not only are these nuclei in a minimum of the p-wave strength function curve but they also have well known resolved s-wave resonance structure[5] with significant interference minima. However, in an experiment one may be faced with the problem of minimizing the nuclear spin-orbit effects but not to such an extent that the $l>0$ scattering is negligible since a small contribution may be desirable as shown in Figs. 3 and 4.

There is a maximum[6] in the s-wave strength function at about $A=50$ and a minimum in the p-wave strength function at about $A=57$. In the range $40<A<50$ $\tan\delta_1{}^0$ has essentially the hard-sphere value. As A goes from 50 to 57 $\tan\delta_1{}^0$ essentially approaches zero. As A goes from 57 to 70 $\tan\delta_1{}^0$ becomes increasingly more positive. These qualitative considerations serve as a rough guide to the importance of the p-wave scattering in the range $40<A<70$.

Experiments[9] may show the enhancement of the optimum angle for polarization. For example, the asymmetry can be measured when a partially polarized beam is scattered near resonance from a nucleus having the required resonance structure. Of course in such experiments the polarization effects under discussion may be smeared out by the energy spread of the incident neutron beam and the angular spread of the scattered neutrons. Hence good resolution in these experimental quantities is required.

[9] The Van de Graaff group at Ohio State University is cooperating with a Battelle group in an attempt to measure these polarization effects associated with resolved s-wave resonances.

ACKNOWLEDGMENTS

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APPENDIX

This Appendix gives a brief derivation of the exact phase-shift result for the additional scattering due to the nuclear spin-orbit interaction of the form given by Eq. (23). The scattering amplitude $f(\theta)$ for a spin-orbit interaction between a spin-zero nucleus and a neutron is well known[10]:

$$kf(\theta)=\sum_l i^{l+1}e^{-il\pi/2}[(2l+1)\pi]^{1/2} \times\left[\left(1-\eta_l{}^+\frac{l+1}{2l+1}-\eta_l{}^-\frac{l}{2l+1}\right)Y_{l,0} -i\frac{\eta_l{}^+-\eta_l{}^-}{2l+1}\sin\theta\frac{dY_{l,0}}{d\cos\theta}\boldsymbol{\sigma}\cdot\mathbf{n}\right], \quad (A1)$$

where $\eta_l{}^+(\eta_l{}^-)$ is the scattering coefficient for $j=l+\frac{1}{2}(j=l-\frac{1}{2})$ waves, j is the total angular momentum quantum number, and $Y_{l,m}$ is the spherical harmonic function. This result is quite easily derived using angular momentum theory and it may be of interest to present this simple derivation.

For convenience assume the z axis for quantization to be directed along the incident plane-wave momentum vector $\hbar\mathbf{k}$. If the incident neutrons have spin projection of m along the z axis the scattered wave ψ_{sc} has the form[11]

$$\psi_{sc}{}^m=-\sum_l i^l[(2l+1)\pi]^{1/2}h_l{}^{(1)}(kr) \times[C(l,\tfrac{1}{2},l+\tfrac{1}{2};0mm)(1-\eta_l{}^+)\mathcal{Y}^l{}_{l+1/2,m} +C(l,\tfrac{1}{2},l-\tfrac{1}{2};0mm)(1-\eta_l{}^-)\mathcal{Y}^l{}_{l-1/2,m}], \quad r>R, \quad (A2)$$

[10] J. Lepore, Phys. Rev. **79**, 137 (1950).

[11] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. 10, pp. 426–429.

where $h_l^{(1)}$ is the spherical Hankel function,[8]

$$\mathcal{Y}_{jm}{}^l=\sum_\nu C(l\tfrac{1}{2}j;\mu\nu m)Y_{l\mu}\chi_{1/2\nu}, \tag{A3}$$

where $Y_{l\mu}$ is the spherical harmonic function, $\chi_{1/2\nu}$ is a spin function $\frac{1}{2}\sigma_z\chi_{1/2\nu}=\nu\chi_{1/2\nu}$, and the C's are Clebsch-Gordan coefficients in Rose's[12] notation.

Asymptotically for large r Eq. (A2) using Eq. (A3) gives

$$\psi_{sc}{}^m\to(e^{ikr}/r)\sum_\nu \varphi_{m\nu}\chi_{1/2\nu}, \tag{A4}$$

where

$$\begin{aligned}\varphi_{m\nu}=k^{-1}\sum_l i^{l+1}e^{-il\pi/2}[(2l+1)\pi]^{1/2}\\ \times[(1-\eta_l{}^+)C(l,\tfrac{1}{2},l+\tfrac{1}{2};0mm)\\ \times C(l,\tfrac{1}{2},l+\tfrac{1}{2},m-\nu,\nu,m)\\ +(1-\eta_l{}^-)C(l,\tfrac{1}{2},l-\tfrac{1}{2};0mm)\\ \times C(l,\tfrac{1}{2},l-\tfrac{1}{2};m-\nu,\nu,m)]Y_{l,m-\nu}.\end{aligned} \tag{A5}$$

Since $m=\pm\frac{1}{2}$ and $\nu=\pm\frac{1}{2}$ it is straightforward to show that this expression is equivalent to the matrix

$$\varphi=k^{-1}\sum_l i^{l+1}e^{-il\pi/2}[(2l+1)\pi]^{1/2}\left(1-\eta_l{}^+\frac{l+1}{2l+1}-\eta_l{}^-\frac{l}{2l+1}-\frac{\eta_l{}^+-\eta_l{}^-}{2l+1}\boldsymbol{\sigma}\cdot\mathbf{l}\right)Y_{l,0}. \tag{A6}$$

Since $\mathbf{l}Y_{l,0}=i\sin\theta(dY_{l,0}/d\cos\theta)\mathbf{n}$ it is easily shown that the result given by Eq. (A1) follows.

The preceding expressions can now be used to derive an exact result for the effect of the spin-orbit interaction of the form given by Eq. (23); i.e., the total potential is given by

$$V(r)=V_0(r)-V_{so}R\delta(r-R)\boldsymbol{\sigma}\cdot\mathbf{l}, \tag{A7}$$

$$\begin{aligned}V_0(r)&=-V_0,\quad r<R,\\ &=0,\quad r>R.\end{aligned} \tag{A8}$$

The total wave function for $r\geq R$ and for $j_z\psi^m=m\psi^m$ is given by

$$\begin{aligned}\psi^m=\sum_l i^l[(2l+1)\pi]^{1/2}\{C(l,\tfrac{1}{2},l+\tfrac{1}{2};0mm)\\ \times[h_l^{(2)}(kr)+\eta_l{}^+h_l^{(1)}(kr)]\mathcal{Y}^l{}_{l+1/2,m}\\ +C(l,\tfrac{1}{2},l-\tfrac{1}{2};0mm)\\ \times[h_l^{(2)}(kr)+\eta_l{}^-h_l^{(1)}(kr)]\mathcal{Y}^l{}_{l-1/2,m}\}.\end{aligned} \tag{A9}$$

The wave function for $V_{so}=0$, $\psi_0{}^m$, can be represented by letting $\eta_l{}^+=\eta_l{}^-=\eta_l{}^0$ in Eq. (A9). Applying the Wronskian theorem[13] then yields

$$\begin{aligned}2i(\eta_l{}^\pm-\eta_l{}^0)=(kR)\alpha b_l{}^\pm\\ \times[h_l^{(2)}(kR)+\eta_l{}^0h_l^{(1)}(kR)]\\ \times[h_l^{(2)}(kR)+\eta_l{}^\pm h_l^{(1)}(kR)],\end{aligned} \tag{A10}$$

where $b_l{}^+=l$, $b_l{}^-=-(l+1)$, and $\alpha=2MV_{so}R^2/\hbar^2$. Thus, Eq. (A10) gives the $\eta_l{}^\pm$ in terms of the $\eta_l{}^0$.

Equation (A10) is more conveniently expressed in terms of phase shifts δ_l, i.e., $\eta_l=e^{i2\delta_l}$. Thus

$$\begin{aligned}j_l(kR)-\tan\delta_l{}^\pm n_l(kR)=[j_l(kR)-\tan\delta_l{}^0n_l(kR)]\\ \times\{1-\alpha(kR)b_l{}^\pm n_l(kR)\\ \times[j_l(kR)-\tan\delta_l{}^0n_l(kR)]\}^{-1},\end{aligned} \tag{A11}$$

where the j_l and n_l are spherical Bessel functions.[14]

For $kR\ll1$, Eq. (A11) reduces to

$$\begin{aligned}\tan\delta_l{}^\pm=\tan\delta_l{}^0\\ \times\{1-\alpha b_l{}^\pm[(2l-1)!!]^2(kR)^{-2l-1}\tan\delta_l{}^0\}^{-1}\end{aligned} \tag{A12}$$

which gives a resonance if the bracketed quantity is zero.

[12] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), Chap. 3, p. 33.

[13] A. Messiah, *Quantum Mechanics* (Interscience Publishers Inc., New York, 1961), Vol. I, Chap. 10, p. 404.

[14] See Ref. 8, Chap. 4, p. 79.